量子性と散逸
— いくつかの新しい視点 —

筑波大学 物理学系

有光 敏彦

(日本語要旨)

非平衡状態にある量子系を扱う正準理論の体系（非平衡 Thermo Field Dynamics [NETFD] と呼ばれている）を解説する。この体系は，非平衡系の統計物理学で重要な４つの視点（Boltzmann 方程式，Fokker-Planck 方程式，Langevin 方程式，確率的 Liouville 方程式によりそれぞれ記述される視点）を，一貫したひとつの土俵で議論することを可能にした。

NETFD 建設の背景と動機，それと「量子コヒーレンスと散逸」に関わる問題を扱う際に有効となる新しい観点に重点をおいて紹介する。また，外場を導入する際に２つの方法があることを示す。NETFD の体系に内在するこの特性は，微視的な視点と巨視的な視点の橋渡しに従来にない観点を与えるであろう。

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有光 敏彦 (Toshihico ARIMITSU)

筑波大学 物理学系
つくば市天王台 1-1-1
Internet: arimitsu@cm.ph.tsukuba.ac.jp

1 Introduction

We will introduce a *canonical formalism* of quantum systems in far-from-equilibrium state, named Non-Equilibrium Thermo Field Dynamics (NETFD) [1]-[6], which provides a unified viewpoint covering whole the aspects in non-equilibrium statistical mechanics, i.e. the Boltzmann, the Fokker-Planck, the Langevin and the stochastic Liouville equations.

Special emphases are put on the background and motivation for the construction of NETFD, and on the new aspects which are raised to solve the problems concerning to *quantum coherence and dissipation* within the framework of NETFD.

In this paper, we will show that there are at least two different ways in the introduction of external force, which are physically acceptable. This notion will open a new aspect for mediating between microscopic (and/or mesoscopic) and macroscopic physics [6].

2 System of Classical Stochastic Differential Equations

Let us remember here the system of stochastic differential equations for classical systems.

The stochastic Liouville equation is given by [16]

$$\frac{\partial}{\partial t}f(u,t) = \Omega(u,t)f(u,t), \qquad \Omega(u,t) = -\frac{\partial}{\partial u}\dot{u}, \tag{1}$$

with the initial condition $f(u,0) = P(u,0)$. The flow $\dot{u}$ in the velocity space is given, for example, by

$$\dot{u} = -\gamma u + \frac{1}{m}R(t). \tag{2}$$

The random force is specified by

$$\Big\langle R(t)\Big\rangle = 0, \qquad \Big\langle R(t_1)R(t_2)\Big\rangle = 2m\gamma T\delta(t_1 - t_2). \tag{3}$$

The Boltzmann constant has been put to equal to unity.

The Langevin equation of the system has the same structure as the flow equation (2), i.e.

$$\dot{u}(t) = -\gamma u(t) + \frac{1}{m}R(t). \tag{4}$$

In precise, the stochastic distribution function $f(u,t)$ means that

$$f(u,t) = f(u,t;\Omega(u,t),P(u,0)). \tag{5}$$

Averaging over all possibilities of $\Omega(u,t)$, we have the usual distribution function $P(u,t)$ as

$$P(u,t)=\Big\langle f(u,t;\Omega(u,t),P(u,0))\Big\rangle, \tag{6}$$

which satisfies the Fokker-Planck equation

$$\frac{\partial}{\partial t}P(u,t)=\frac{\partial}{\partial u}\gamma\left(u+\frac{T}{m}\frac{\partial}{\partial u}\right)P(u,t). \tag{7}$$

We see that it has a stationary solution

$$P_{eq}(u)=C\exp\left(-\frac{m}{2T}u^2\right). \tag{8}$$

Note that $f(u,t)$ satisfies the conservation of probability within the velocity space:

$$\int du\ f(u,t)=\text{constant}=1, \tag{9}$$

which can be seen from (1). Note also that the Langevin equation (4) does *not* contain the diffusion term. This is a Stratonovich type stochastic differential equation [17]. One can proceed calculation as if the stochastic function $u(t)$ were an analytic one. The fluctuation-dissipation theorem of the second kind is introduced in order that the Langevin equation (4) is consistent with the Fokker-Planck equation (7).

3 Liouville Equation

The most basic characteristics of the Liouville equation

$$\frac{\partial}{\partial t}\rho(t)=-iL\rho(t), \tag{10}$$

may be summarized as follows:

D1. The hermiticy of the Liouville operator iL: $(iL)^\dagger=iL$.

D2. The conservation of probability (tr $\rho=1$): tr $LX=0$.

D3. The hermiticy of the density operator: $\rho^\dagger(t)=\rho(t)$.

The expectation value of an observable A is given by $\langle A\rangle_t=\mathrm{tr}\ A\rho(t)$. Substitution of the formal solution $\rho(t)=\mathrm{e}^{-iLt}\rho(0)$ gives us a Heisenberg operator

$$A(t)=\mathrm{e}^{iLt}A\mathrm{e}^{-iLt}, \tag{11}$$

through the process $\langle A\rangle_t=\mathrm{tr}\ A\mathrm{e}^{-iLt}\rho=\mathrm{tr}\ \mathrm{e}^{iLt}A\mathrm{e}^{-iLt}\rho=\mathrm{tr}\ A(t)\rho$. The Heisenberg operator satisfies the Heisenberg equation

$$\frac{dA(t)}{dt}=i[L,\ A(t)]. \tag{12}$$

4 Technical Basics of NETFD

The minimal technical tools needed for NETFD is the following.

Tool 1. Any operator A in NETFD is accompanied by its partner (tilde) operator $\tilde{A}$. The tilde conjugation $\sim$ is defined by $(A_1A_2)^\sim = \tilde{A}_1\tilde{A}_2$, $(c_1A_1+c_2A_2)^\sim = c_1^*\tilde{A}_1+c_2^*\tilde{A}_2$, $(\tilde{A})^\sim = A$, and $(A^\dagger)^\sim = \tilde{A}^\dagger$.

Tool 2. The tilde and non-tilde operators at an equal time are commutative: $[A,\ \tilde{B}] = 0$.

Tool 3. The tilde and non-tilde operators are mutually related through the bra-vacuum $\langle 1|$ as $\langle 1|A^\dagger = \langle 1|\tilde{A}$.

Corresponding to the characteristics of the Liouville operator given in the previous section, the Schrödinger equation ($\hbar = 1$) within NETFD

$$\frac{\partial}{\partial t}|0(t)\rangle = -i\hat{H}\ |0(t)\rangle, \tag{13}$$

has the following basic characters:

B1. The characteristics named ***tildian***:

$$\left(i\hat{H}\right)^\sim = i\hat{H}. \tag{14}$$

The tildian hat-Hamiltonian is not necessarily hermitian operator.

B2. The hat-Hamiltonians have zero eigenvalue for the thermal bra-vacuum:

$$\langle 1|\hat{H} = 0. \tag{15}$$

This is the manifestation of the conservation of probability, i.e. $\langle 1|0(t)\rangle = 1$.

B3. The thermal vacuums $\langle 1|$ and $|0\rangle$ are ***tilde invariant***:

$$\langle 1|^\sim = \langle 1|, \quad |0\rangle^\sim = |0\rangle, \tag{16}$$

and are normalized as $\langle 1|0\rangle = 1$.

The Heisenberg equation within NETFD is given by

$$\frac{d}{dt}A = i[\hat{H},\ A]. \tag{17}$$

Let us see the meaning of the thermal vacuum $|0(t)\rangle$ by representing it in terms of the orthonormal and complete set $|m,\tilde{n}\rangle$ satisfying $\langle m,\tilde{n}|m',\tilde{n}'\rangle = \delta_{m,m'}\delta_{n,n'}$, $\sum_{m,n}|m,\tilde{n}\rangle\langle m,\tilde{n}| = 1$. The state behaves as $|m,\tilde{n}\rangle^\sim = |n,\tilde{m}\rangle$ under a tilde conjugation.

The coefficients $P_{n,m}(t)$ in the expansion

$$|0(t)\rangle = \sum_{n,m} P_{n,m}(t)|n,\tilde{m}\rangle, \qquad \langle 1| = \sum_n \langle n,\tilde{n}|, \tag{18}$$

satisfy the normalization

$$1 = \langle 1|0(t)\rangle = \sum_k \sum_{n,m} P_{n,m}(t)\langle k,\tilde{k}|n,\tilde{m}\rangle = \sum_k P_{k,k}(t), \tag{19}$$

and $P^*_{m,n}(t) = P_{n,m}(t)$, which can be derived by the tilde-invariance of the ket-vacuum as

$$|0(t)\rangle^\sim = \sum_{n,m} P^*_{n,m}(t)|n,\tilde{m}\rangle^\sim = \sum_{n,m} P^*_{n,m}(t)|m,\tilde{n}\rangle = \sum_{m,n} P^*_{m,n}(t)|n,\tilde{m}\rangle = |0(t)\rangle. \tag{20}$$

5 Quantum Fokker-Planck Equation

5.1 A Semi-Free System

The hat-Hamiltonian for a bosonic semi-free field is bi-linear in a, $a^\dagger$ and their tilde-conjugates, and is invariant under the phase transformation $a \longrightarrow ae^{i\theta}$:

$$\begin{aligned}\hat{H} &= \omega\left(a^\dagger a - \tilde{a}^\dagger \tilde{a}\right) - i\kappa\left[(1+2\bar{n})\left(a^\dagger a + \tilde{a}^\dagger \tilde{a}\right) - 2(1+\bar{n})\, a\tilde{a} - 2\bar{n} a^\dagger \tilde{a}^\dagger\right] - i2\kappa\bar{n} \\ &= (\omega - i\kappa)\, \bar{a}^\mu a^\mu - i2\kappa \bar{a}^\mu \bar{n}^{\mu\nu} a^\nu + \omega + i\kappa, \end{aligned} \tag{21}$$

where we introduced the thermal doublet notations, $a^{\mu=1} = a$, $a^{\mu=2} = \tilde{a}^\dagger$, $\bar{a}^{\mu=1} = a^\dagger$, $\bar{a}^{\mu=2} = -\tilde{a}$, and

$$\bar{n}^{\mu\nu} = \begin{pmatrix} \bar{n} & -\bar{n} \\ 1+\bar{n} & -(1+\bar{n}) \end{pmatrix}, \qquad \bar{n} = \frac{1}{e^{\omega/T} - 1}. \tag{22}$$

T represents the temperature of environment. The boson operators a etc. satisfy the canonical commutation relations:

$$[a,\ a^\dagger] = 1, \qquad [\tilde{a},\ \tilde{a}^\dagger] = 1. \tag{23}$$

The Fokker-Planck equation of the system is given by

$$\frac{\partial}{\partial t}|0(t)\rangle = -i\hat{H}|0(t)\rangle, \tag{24}$$

with an initial ket-thermal vacuum, $|0\rangle \ = \ |0(0)\rangle$, satisfying

$$a|0\rangle = f\tilde{a}^\dagger|0\rangle, \tag{25}$$

with $f = n/(1{+}n)$, $n = n(0)$. The Fokker-Planck equation (24) has the form of the Schrödinger equation. It was noticed first by Crawford [7] that the introduction of two kinds of operators for each operator enables us to handle the Liouville equation as the Schrödinger equation.

With the help of the Fokker-Planck equation (24), we see that the one-particle distribution function $n(t) = \langle 1|a^\dagger a|0(t)\rangle$ satisfies the Boltzmann equation

$$\frac{d}{dt}n(t) = -2\kappa\,[n(t) - \bar{n}]\,. \tag{26}$$

5.2 Irreversibility

One of the observables appropriate to check the irreversibility of the system is the Boltzmann entropy introduced by

$$\mathcal{S}(t) = -\left\{n(t)\ln n(t) - [1+n(t)]\ln[1+n(t)]\right\}. \tag{27}$$

This entropy is also appropriate for thermodynamical argument within quantum field theory, since it is related to the particle aspect which is essential to the quantum field theoretical representation in terms of asymptotic fields.

The heat change $d'Q$ of the relevant system is given by $d'Q = \omega dn$ which is related to the extrinsic entropy by the relation $d\mathcal{S}_e = d'Q/T$ where T represents the temperature of the thermal reservoir. Here, we decomposed the change of the entropy $d\mathcal{S}$ into two parts as $d\mathcal{S} = d\mathcal{S}_e + d\mathcal{S}_i$.

The *entropy production rate* dS_i/dt of the present system is then given by [10]

$$\frac{dS_i}{dt} = \frac{dS}{dt} - \frac{dS_e}{dt} = 2\kappa\,[n(t) - \bar{n}]\ln\frac{n(t)[1+\bar{n}]}{\bar{n}[1+n(t)]} \geq 0. \tag{28}$$

The last inequality represents the irreversibility of the system (the second law of thermodynamics).

There can be other entropy measuring the degree of *mixing* of the states of the relevant system. That is defined by means of the *norm* of the thermal ket-vacuum [3]: $\Omega(t) = \langle 0(t)|0(t)\rangle = \left\| |0(t)\rangle \right\|^2$, with $\langle 0(t)| = |0(t)\rangle^\dagger$ in the form

$$\mathcal{S}^{(N)}(t) = -\frac{1}{2}\ln\ \Omega(t). \tag{29}$$

When the initial state is a pure state, say the ground state specified by $a|0\rangle = 0$ or $f = 0$, the entropy production rate is given by

$$\frac{d\mathcal{S}^{(N)}}{dt} = \frac{2\kappa\bar{n}\mathrm{e}^{-2\kappa t}}{1 + 2\bar{n}\left(1 - \mathrm{e}^{-2\kappa t}\right)} \geq 0. \tag{30}$$

We can solve this with the initial condition $\mathcal{S}^{(N)} = 0$ as

$$\mathcal{S}^{(N)}(t) = \frac{1}{2}\ln\ [1 + 2n(t)], \tag{31}$$

with the one-particle distribution function $n(t) = \bar{n}\left(1 - \mathrm{e}^{-2\kappa t}\right)$.

6 Inclusion of External Fields

There are two physically admissible forms of interaction hat-Hamiltonian which take into account the effect of an external field. One is hermitian, and the other is non-hermitian [6].

6.1 Hermitian Interaction Hat-Hamiltonian

The simplest hermitian interaction hat-Hamiltonian is

$$\hat{H}_t' = H_t' - \tilde{H}_t', \qquad H_t' = i\left[a^\dagger b(t) - b^\dagger(t) a\right], \tag{32}$$

where $b(t)$ and $b^\dagger(t)$ are operators in the external system, being commutative with the operators a, $a^\dagger$ of the relevant system, and satisfy

$$\langle| b^\dagger(t) = \langle| \tilde{b}(t). \tag{33}$$

Applying the bra-vacuum $\langle 1|$ of the relevant system, we obtain from (32)

$$\langle 1|\hat{H}_t' = -i\langle 1|\left[a\beta^{\natural}(t) + a^\dagger\tilde{\beta}^{\natural}(t)\right], \tag{34}$$

with $\beta^{\natural}(t) = b^\dagger(t) - \tilde{b}(t)$ which annihilates the bra-vacuum $\langle|$ of the external system: $\langle|\beta^{\natural}(t) = 0$. Applying $\langle|$, we see that $\langle\!\langle 1|\hat{H}_t' = 0$, with $\langle\!\langle 1| = \langle| \cdot \langle 1|$.

The above investigation shows that a simple introduction of an hermitian interaction hat-Hamiltonian violates the conservation of probability within the relevant system: We need to apply $\langle\!\langle 1|$ instead of $\langle 1|$ to

$$\frac{\partial}{\partial t}|0(t)\rangle = -i\left(\hat{H} + \hat{H}_t'\right)|0(t)\rangle, \tag{35}$$

in order to observe the conservation of probability.

6.2 Non-Hermitian Interaction Hat-Hamiltonian

The requirement that the Schrödinger equation

$$\frac{\partial}{\partial t}|0(t)\rangle = -i\left(\hat{H} + \hat{H}_t''\right)|0(t)\rangle, \tag{36}$$

satisfies the conservation of probability within the relevant system reduces to

$$\langle 1|\hat{H}_t'' = 0, \tag{37}$$

leading

$$\hat{H}_t'' = i\left[\alpha^{\text{?}}\beta(t) + \text{t.c.}\right], \tag{38}$$

where $\alpha^{\text{?}} = a^\dagger - \tilde{a}$, $\beta(t) = \mu b(t) + \nu\tilde{b}^\dagger(t)$, with $\mu + \nu = 1$. Note the commutation relation

$$[\beta(t),\ \beta^{\text{?}}(t)] = 1. \tag{39}$$

The interaction hat-Hamiltonian (38) is tildian but *not* hermitian.

6.3 Relation between the Two Interaction Hat-Hamiltonians

The above two hat-Hamiltonians are related with each other by

$$\hat{H}_t' = \hat{H}_t'' - i\left[\alpha\beta^{\text{?}}(t) + \text{t.c.}\right], \tag{40}$$

with $\alpha = \mu a + \nu\tilde{a}^\dagger$. Note the commutation relation

$$[\alpha,\ \alpha^{\text{?}}] = 1. \tag{41}$$

7 $\hat{S}$-matrix

The solution of the Fokker-Planck equation

$$\frac{\partial}{\partial t}|0(t)\rangle = -i\hat{H}_t^{tot}|0(t)\rangle, \qquad \hat{H}_t^{tot} = \hat{H}_t + \hat{H}_t', \tag{42}$$

is given by

$$|0(t)\rangle = \hat{V}(t)\hat{S}(t,t_0)\hat{V}^{-1}(t_0)|0(t_0)\rangle, \tag{43}$$

with the help of the $\hat{S}$-*matrix*

$$\hat{S}(t,t_0) = \hat{S}(t)\hat{S}^{-1}(t_0). \tag{44}$$

Here, $\hat{S}(t)$ is specified by

$$\frac{d}{dt}\hat{S}(t) = -i\hat{H}'(t)\hat{S}(t), \qquad \hat{S}(t_0) = 1. \tag{45}$$

with

$$\hat{H}'(t) = \hat{V}^{-1}(t)\hat{H}_t'\hat{V}(t). \tag{46}$$

7.1 Interaction Representation

Let us introduce operators in the interaction representation, named semi-free operators, by

$$a(t)=\hat{V}^{-1}(t)a\hat{V}(t), \qquad \tilde{a}^{\dagger\!\!\dagger}(t)=\hat{V}^{-1}(t)\tilde{a}^{\dagger}\hat{V}(t), \tag{47}$$

where

$$\frac{d}{dt}\hat{V}(t)=-i\hat{H}\hat{V}(t), \qquad \hat{V}(0)=1. \tag{48}$$

The semi-free operators preserve the canonical commutation relation:

$$[a(t),\ a^{\dagger}(t)]=1, \qquad [\tilde{a}(t),\ \tilde{a}^{\dagger}(t)]=1. \tag{49}$$

The $\hat{S}$-matrix satisfies $\langle 1|\hat{S}(t,t_0) = \langle 1|$. This is a manifestation of the conservation of probability in whole the system, $\langle 1|0(t)\rangle = 1$.

Expanding the $\hat{S}$-matrix with respect to the order of $\hat{H}_t'$ as

$$\hat{S}(t,t_0)=\sum_{n=0}^{\infty}\hat{S}^{(n)}(t,t_0), \tag{50}$$

we can deal with any order of processes induced by $\hat{H}_t'$ (see, for example, [11]).

With the $\hat{S}$-matrix, the generating functional for the system is given by

$$Z[K,\tilde{K}]=\langle 1|\hat{S}(\bar{t},0)|0\rangle, \tag{51}$$

and has the expressions [12]

$$\begin{aligned} Z[K,\tilde{K}] &= \exp\left[-i\int_0^{\bar{t}}dt\int_0^{\bar{t}}dt'\bar{K}_\gamma(t)^\mu\mathcal{G}(t,t')^{\mu\nu}K_\gamma(t')^\nu\right] \\ &= \exp\left[-i\int_0^{\bar{t}}dt\int_0^{\bar{t}}dt'\bar{K}(t)^\mu G(t,t')^{\mu\nu}K(t')^\nu\right]. \end{aligned} \tag{52}$$

This expression for an open system was derived first by Schwinger by means of the closed-time path method [13] (see also [14, 15]).

8 Stochastic Semi-Free Hat-Hamiltonian

8.1 Quantum Stochastic Liouville Equations

8.1.1 Ito Type

The hat-Hamiltonian for the *stochastic semi-free* field is bi-linear in a, $a^\dagger$, $dF(t)$, $dF^\dagger(t)$ and their tilde conjugates, and is invariant under the phase transformation $a \to ae^{i\theta}$, and $dF(t) \to dF(t)\ e^{i\theta}$. Here, a, $a^\dagger$ and their tilde conjugates are stochastic operators of a relevant system satisfying the canonical commutation relation[1]

$$[a,\ a^\dagger]=1, \qquad [\tilde{a},\ \tilde{a}^\dagger]=1, \tag{53}$$

[1] We use the same notation a etc. for the stochastic semi-free operators as those for the coarse grained semi-free operators. We expect that there will be no confusion between them.

whereas $dF(t)$, $dF^{\dagger}(t)$ and their conjugates are random force operators.

The tilde and non-tilde operators are related with each other by the relations

$$\langle 1|a^{\dagger} = \langle 1|\tilde{a}, \qquad \langle|dF^{\dagger}(t) = \langle|d\tilde{F}(t), \tag{54}$$

where $\langle 1|$ and $\langle|$ are respectively the thermal bra-vacuum of the relevant system and of the random force.

We will employ the characteristics of the stochastic Liouville equation of classical systems to quantum cases, i.e., the stochastic distribution function satisfies the conservation of probability within the phase space of a relevant system (see the previous section). This means in NETFD that $\langle 1|0_f(t)\rangle = 1$, leading to

$$\langle 1|\hat{\mathcal{H}}_{f,t}dt = 0. \tag{55}$$

Here the thermal bra-vacuum $\langle 1|$ is of the relevant system.

From the investigation in subsection 6.2 (cf. (38)), we know that the required form of the hat-Hamiltonian should be

$$\hat{\mathcal{H}}_{f,t}dt = \hat{H}dt + i\left\{\alpha^{\natural}dW(t) + \text{t.c.}\right\}, \tag{56}$$

where $\hat{H}$ is given by (21), i.e. $\hat{H} = \hat{H}_S + i\hat{\Pi}$, $\hat{\Pi} = -\kappa\left(\alpha^{\natural}\alpha + \text{t.c.}\right) + 2\kappa\left[\bar{n}+\nu\right]\alpha^{\natural}\tilde{\alpha}^{\natural}$. We introduced a set of canonical stochastic operators[2] $\alpha = \mu a + \nu\tilde{a}^{\dagger}$, $\alpha^{\natural} = a^{\dagger} - \tilde{a}$, with $\mu + \nu = 1$, which satisfy the commutation relation $[\alpha,\ \alpha^{\natural}] = 1$.

The random force operators $dW(t), d\tilde{W}(t)$ are of the quantum stochastic Wiener process satisfying

$$\left\langle dW(t)\right\rangle = \left\langle d\tilde{W}(t)\right\rangle = 0, \qquad \left\langle dW(t)dW(s)\right\rangle = \left\langle d\tilde{W}(t)d\tilde{W}(s)\right\rangle = 0, \tag{57}$$

$$\left\langle dW(t)d\tilde{W}(s)\right\rangle = \left\langle d\tilde{W}(s)dW(t)\right\rangle = 2\kappa\left[\bar{n}+\nu\right]\delta(t-s)dtds, \tag{58}$$

with $\langle\cdots\rangle = \langle|\cdots|\rangle$, where the random force operator $dW(t)$ is defined by

$$dW(t) = \mu dF(t) + \nu d\tilde{F}^{\dagger}(t). \tag{59}$$

The *original* random force operators $dF(t)$ and $dF^{\dagger}(t)$ are of the *stationary* Gaussian white process, which is defined by $\left\langle dF(t)\right\rangle = \left\langle d\tilde{F}(t)\right\rangle = \left\langle dF^{\dagger}(t)\right\rangle = \left\langle d\tilde{F}^{\dagger}(t)\right\rangle = 0$,

$$\left\langle dF^{\dagger}(t)dF(s)\right\rangle = 2\kappa\bar{n}\delta(t-s)dtds, \qquad \left\langle dF(t)dF^{\dagger}(s)\right\rangle = 2\kappa\left[\bar{n}+1\right]\delta(t-s)dtds, \tag{60}$$

and zero for other combinations. The one-particle distribution function $n(t) = \langle\!\langle 1|a^{\dagger\!\dagger}(t)a(t)|0_f\rangle\!\rangle$ satisfies the Boltzmann equation (26) where $\langle\!\langle\cdots\rangle\!\rangle = \langle|\langle 1|\cdots|0\rangle|\rangle$, which means to take both random average and vacuum expectation.

Taking the random average of the stochastic Liouville equation

$$d|0_f(t)\rangle = -i\hat{\mathcal{H}}_{f,t}dt|0_f(t)\rangle, \tag{61}$$

we see that it reduces to the Fokker-Planck equation (24) with $|0(t)\rangle = \left\langle|0_f(t)\rangle\right\rangle$, if the condition $\left\langle\left\{\alpha^{\natural}dW(t) + \text{t.c.}\right\}|0_f(t)\rangle\right\rangle = 0$ satisfies. This indicates that the multiplication should be of the Ito type [18]. The random force operator $dW(t)$ does not correlate with the quantities at time t.

[2]The expression of $\hat{\Pi}$ was given here by means of a set of canonical stochastic operators α, $\alpha^{\natural}$ and their tilde conjugates.

8.1.2 Stratonovich Type

By making use of the relation between the Ito and the Stratonovich stochastic multiplications, we can rewrite the Ito type stochastic Liouville equation (61) into the Stratonovich type as follows.

$$d|0_f(t)\rangle = -i\hat{H}_{f,t}dt \circ |0_f(t)\rangle, \tag{62}$$

with

$$\hat{H}_{f,t}dt = \hat{H}_S dt + \left[\alpha^{\dagger}\left(id\alpha + [\hat{H}_{S,t}dt,\ \alpha]\right) - \text{t.c.}\right], \tag{63}$$

where the flow operators $d\alpha$ and $d\tilde{\alpha}$ are specified respectively by

$$d\alpha = i[\hat{H}_S dt,\ \alpha] - \kappa\alpha dt + dW(t), \tag{64}$$

and its tilde conjugate. We introduced the symbol $\circ$ in order to indicate the Stratonovich stochastic multiplication [17].

We can derive the Fokker-Planck equation (24) by taking the random average of the Stratonovich stochastic Liouville equation (62).

8.2 Stochastic Semi-Free Operators

The stochastic semi-free operators are defined by

$$a(t) = \hat{V}_f^{-1}(t) a \hat{V}_f(t), \qquad \tilde{a}^{\dagger\dagger}(t) = \hat{V}_f^{-1}(t)\tilde{a}^{\dagger}\hat{V}_f(t), \tag{65}$$

where

$$d\hat{V}_f(t) = -i\hat{\mathcal{H}}_{f,t}dt\ \hat{V}_f(t), \tag{66}$$

with $\hat{V}_f(0) = 1$. Here, it is assumed that, at $t = 0$, the relevant system starts to contact with the irrelevant system representing the stochastic process described by the random force operators $dF(t)$, etc. defined in (60).[3]

The semi-free operators (65) keep the equal-time canonical commutation relation:

$$[a(t),\ a^{\dagger\dagger}(t)] = 1, \qquad [\tilde{a}(t),\ \tilde{a}^{\dagger\dagger}(t)] = 1, \tag{67}$$

and satisfy **Tool 3**: $\langle 1|a^{\dagger\dagger}(t) = \langle 1|\tilde{a}(t)$. The tildian nature $\left(i\hat{\mathcal{H}}_{f,t}dt\right)^{\sim} = i\hat{\mathcal{H}}_{f,t}dt$, (see **B1**) is consistent with the definition (65) of the semi-free operators. Since the tildian hat-Hamiltonian $\hat{\mathcal{H}}_{f,t}dt$ is not necessarily hermite, we introduced the symbol $\dagger\dagger$ in order to distinguish it from the hermite conjugation $\dagger$. However, we will use $\dagger$ instead of $\dagger\dagger$, for simplicity, unless it is confusing.

The stochastic semi-free operators and the random force operators satisfy the orthogonality $\left\langle a(t)d\mathcal{F}^{\dagger}(t)\right\rangle = 0$, etc., where the random force operator $d\mathcal{F}^{\dagger}(t)$ in the *Heisenberg* representation[4] is defined by $d\mathcal{F}^{\dagger}(t) = \hat{V}_f^{-1}(t)dF^{\dagger}(t)\hat{V}_f(t)$.

[3]Within the formalism, the random force operators $dF(t)$ and $dF^{\dagger}(t)$ are assumed to commute with any relevant system operator A in the Schrödinger representation: $[A,\ dF(t)] = [A,\ dF^{\dagger}(t)] = 0$.

[4]It can be the interaction representation when one includes non-linear terms in the hat-Hamiltonian, and performs a perturbational calculation. As we are dealing with only the semi-free case in this section, we call the representation as the Heisenberg one.

8.3 Quantum Langevin Equations

8.3.1 Stratonovich type

For the dynamical quantity $A(t) = \hat{V}_f^{-1}(t)A\hat{V}_f(t)$, the quantum Langevin equation of the Stratonovich type is given by the stochastic Heisenberg equation as [19]

$$dA(t) = i[\hat{H}_f(t)dt \mathbin{\circ\!\!,} A(t)] \tag{68}$$

$$= i[\hat{H}_S(t),\ A(t)]dt + \kappa\left\{[\alpha^{\dagger}(t)\alpha(t),\ A(t)] + [\tilde{\alpha}^{\dagger}(t)\tilde{\alpha}(t),\ A(t)]\right\}dt$$
$$-\left\{[\alpha^{\dagger}(t),\ A(t)]\circ dW(t) + [\tilde{\alpha}^{\dagger}(t),\ A(t)]\circ d\tilde{W}(t)\right\}, \tag{69}$$

where $\hat{H}_f(t) = \hat{V}_f^{-1}(t)\hat{H}_{f,t}\hat{V}_f(t)$, $\hat{H}_S(t) = \hat{V}_f^{-1}(t)\hat{H}_{S,t}\hat{V}_f(t)$, $[X(t) \mathbin{\circ\!\!,} Y(t)] = X(t)\circ Y(t) - Y(t)\circ X(t)$, for arbitrary operators $X(t)$ and $Y(t)$. Use has been made of the fact that

$$\hat{V}_f^{-1}(t)dW(t)\hat{V}_f(t) = dW(t), \tag{70}$$

since the random force operator $dW(t)$ is commutative with $\hat{V}_f(t)$.

8.3.2 Ito type

By means of the connection formula between the Ito and the Stratonovich products, we can derive the quantum Langevin equation of the Ito type from that of the Stratonovich type (69) as

$$dA(t) = i[\hat{\mathcal{H}}_f(t)dt,\ A(t)]$$
$$+\left\{\alpha^{\dagger}(t)[\tilde{\alpha}^{\dagger}(t),\ A(t)] + \tilde{\alpha}^{\dagger}(t)[\alpha^{\dagger}(t),\ A(t)]\right\}dW(t)d\tilde{W}(t) \tag{71}$$

$$= i[\hat{H}_S(t),\ A(t)]dt + \kappa\left\{[\alpha^{\dagger}(t)\alpha(t),\ A(t)] + [\tilde{\alpha}^{\dagger}(t)\tilde{\alpha}(t),\ A(t)]\right\}dt$$
$$+2\kappa(\bar{n}+\nu)[\tilde{\alpha}^{\dagger}(t),\ [\alpha^{\dagger}(t),\ A(t)]]dt$$
$$-\left\{[\alpha^{\dagger}(t),\ A(t)]dW(t) + [\tilde{\alpha}^{\dagger}(t),\ A(t)]d\tilde{W}(t)\right\}. \tag{72}$$

Since (72) is the time-evolution equation for any relevant stochastic operator $A(t)$, it is *Ito's formula* for quantum systems.

Putting a and $\tilde{a}^{\dagger}$ for A, we see that both (69) and (72) reduce to

$$d\alpha(t) = i[\hat{H}_S(t)dt,\ \alpha(t)] - \kappa\alpha(t)dt + dW(t), \tag{73}$$
$$d\alpha^{\dagger}(t) = i[\hat{H}_S(t)dt,\ \alpha^{\dagger}(t)] + \kappa\alpha^{\dagger}(t)dt. \tag{74}$$

The formal structures of (73) is the same as the flow operator (64) appeared in $\hat{H}_{f,t}$ of (63).

In the Langevin equation approach, the dynamical behavior of systems is specified when one characterizes the correlations of random forces. The quantum Langevin equation is the equation in the Heisenberg representation, therefore the characterization of random force operators should be performed in this representation. This cannot be done in terms of $d\mathcal{F}(t)$ etc., since the information of the stochastic process is masked by the dynamics generated by $\hat{H}_f(t)$ in these operators. Whereas, the specification of the correlation between $dW(t)$ etc. directly characterizes the stochastic process owing to the relations in (70).

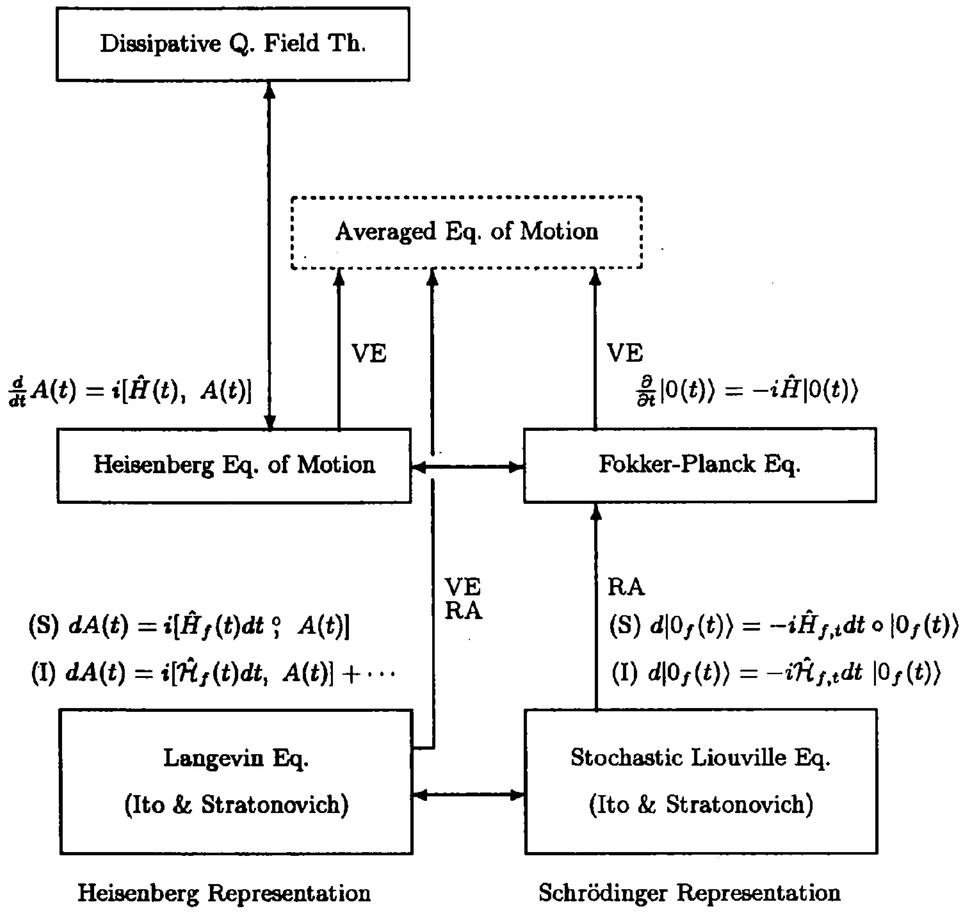

Figure 1: Structure of the Formalism. RA stands for the random average. VE stands for the vacuum expectation. (I) and (S) indicate Ito and Stratonovich types, respectively.

9 Discussions

We showed that, by the success of the formulation of NETFD, it has come to be possible to investigate dissipative quantum systems systematically upon a unified stand point. It should be noted that the discovery of the stochastic Liouville equation is the key point for the construction of whole the unified quantum canonical formalism (see Fig. 1).

The framework of NETFD has a potential to open various new prospective aspects. We are closing the paper by listing here some of them in the following.

9.1 Consistency with the Classical Framework

As was shown in this paper, whole the structure of the formulation was constructed in the manner being consistent with that of classical one: The stochastic Liouville equation satisfies the conservation of probability within the relevant system; The Stratonovich stochastic differential equation contains the relaxation generator but does not contain the diffusion generator, whereas the Ito equation does both generators.

Note that this consistency is violated for the approach with the *hermitian* interaction hat-Hamiltonian as will be shown in the following. However, this would be important when we investigate the relation of NETFD with the mathematical formulations [22]-[33], which is an interesting future problem.

Unitary Time-Generation

If we take the hermitian hat-Hamiltonian of the type (32) for the interaction between the relevant system and the random force, the time-evolution generator for the stochastic Liouville equation (62) of the Stratonovich type becomes

$$\hat{H}_{f,t}dt = \hat{H}_S dt + i\left[\left(a^\dagger - \tilde{a}\right) dW(t) + \text{t.c.}\right] - i\left[\left(\mu a + \nu \tilde{a}^\dagger\right) dW^{\text{♀}}(t) + \text{t.c.}\right]. \tag{75}$$

In addition to the random force operators $dW(t)$ and its tilde conjugate, we need to introduce

$$dW^{\text{♀}}(t) = dF^\dagger(t) - d\tilde{F}(t), \tag{76}$$

and its tilde conjugate which annihilate the ket-vacuum $\langle|$:

$$\langle| dW^{\text{♀}}(t) = 0, \qquad \langle| d\tilde{W}^{\text{♀}}(t) = 0. \tag{77}$$

The new random force operators satisfy the correlations

$$\left\langle dW^{\text{♀}}(t)\right\rangle = \left\langle d\tilde{W}^{\text{♀}}(t)\right\rangle = 0, \qquad \left\langle dW^{\text{♀}}(t)dW(s)\right\rangle = \left\langle d\tilde{W}^{\text{♀}}(t)d\tilde{W}(s)\right\rangle = 0, \tag{78}$$

$$\left\langle dW(t)dW^{\text{♀}}(s)\right\rangle = \left\langle d\tilde{W}(t)d\tilde{W}^{\text{♀}}(s)\right\rangle = 2\kappa\delta(t-s)dt\, ds. \tag{79}$$

In this case, as the hat-Hamiltonian is hermitian:

$$\left(\hat{H}_{f,t}dt\right)^\dagger = \hat{H}_{f,t}dt, \tag{80}$$

the stochastic time-evolution generator $\hat{V}_f(t)$ satisfying

$$d\hat{V}_f(t) = -i\hat{H}_{f,t} \circ \hat{V}_f(t), \tag{81}$$

becomes unitary:

$$\hat{V}_f^\dagger(t) = \hat{V}_f^{-1}(t), \tag{82}$$

within the Stratonovich calculation.

Starting with this hermitian hat-Hamiltonian, we can proceed just the same way as in the previous section for the rest of the formulation to construct a unified framework of the stochastic differential equations. We see that the equation of motion for the ket-vector $\langle\!\langle 1|A(t)$ reduces to (84). Therefore, the averaged equation of motion reduces also to (85) in both cases [6].

9.2 Equation of Motion for the Ket-Vectors

Applying the random force bra-vacuum $\langle 1|$ to the Ito type Langevin equation (72), we have

$$\begin{aligned} d\langle 1|A(t) &= i\langle 1|[H_S(t),\ A(t)]dt + \kappa\langle 1|A(t)\left[\alpha^{\text{♀}}\alpha + \text{t.c.}\right]dt + 2\kappa\bar{n}\langle 1|A(t)\alpha^{\text{♀}}(t)\tilde{\alpha}^{\text{♀}}(t)dt \\ &\quad + \langle 1|A(t)\left[\alpha^{\text{♀}}(t)dW(t) + \text{t.c.}\right]. \end{aligned} \tag{83}$$

Applying further the bra-vacuum $\langle|$ to (83), we can derive the stochastic equation of motion of Ito type for the bra-vector state $\langle\!\langle 1|A(t)$ in the form

$$\begin{aligned} d\langle\!\langle 1|A(t) &= i\langle\!\langle 1|[H_S(t),\ A(t)]dt + \kappa\left\{\langle\!\langle 1|[a^\dagger(t),\ A(t)]a(t) + \langle\!\langle 1|a^\dagger(t)[A(t),\ a(t)]\right\}dt \\ &\quad + 2\kappa\bar{n}\langle\!\langle 1|[a(t),\ [A(t),\ a^\dagger(t)]]dt \\ &\quad + \langle\!\langle 1|[A(t),\ a^\dagger(t)]dF(t) + \langle\!\langle 1|[a(t),\ A(t)]dF^\dagger(t), \end{aligned} \tag{84}$$

where we used the property $\langle|dW(t) = \langle|dF(t)$ and $\langle|d\tilde{W}(t) = \langle|dF^{\dagger}(t)$. This equation of motion for the bra-vector state may be intimately related with the Langevin equations given by Gardiner and Collett [20].

Putting the random force ket-vacuum $|\rangle$ and the ket-vacuum $|0\rangle$ of the relevant system to (84), we obtain the equation of motion for the expectation value of an arbitrary operator $A(t)$ of the relevant system as

$$\frac{d}{dt}\langle\langle A(t)\rangle\rangle = i\,\langle\langle[H_S(t),\ A(t)]\rangle\rangle + \kappa\left(\langle\langle[a^{\dagger}(t),\ A(t)]a(t)\rangle\rangle + \langle\langle a^{\dagger}(t)[A(t),\ a(t)]\rangle\rangle\right) \\ +2\kappa\bar{n}\langle\langle[a(t),\ [A(t),\ a(t)^{\dagger}]]\rangle\rangle. \tag{85}$$

This is the exact equation of motion for systems with linear-dissipative coupling to reservoir, which can be also derived by means of Fokker-Planck equation (24). Note that (85) was derived for general $\hat{H}_S$ including non-linear interaction terms within the conventional treatment [21].

The correspondence of the equation of motion for the ket-vector $\langle\langle 1|A(t)$, (84), to the Langevin equation derived by Gardiner and Collet [20], is an attractive future problem. For spin systems, there is a similar correspondence between the equation of motion for ket-vector and the Langevin equation derived by Shibata and Hashitsume [34]. An investigation related to these correspondences may give us a deeper insight for the derivation of the stochastic differential equations from a microscopic point of view.

9.3 Dissipation as a Condensate

The problem of the representation space within NETFD, e.g. a foundation of the concept of the spontaneous creation of dissipation, should be studied with the help of the rigged Hilbert space [35]. A system traverses among unitarily inequivalent representation spaces in approaching to a thermal equilibrium (or stationary) state [36]. With an appropriate measure, the distribution of the unitarily inequivalent representation spaces may provide us with a new concept of entropy. In this connection, we expect that the mathematical approach by Obata [31]-[33] will provide us with an attractive view point.

The annihilation and creation operators, $\gamma^{\mu=1} = \gamma_t$, $\gamma^{\mu=2} = \tilde{\gamma}^{\dagger}$, $\bar{\gamma}^{\mu=1} = \gamma^{\dagger}$, $\bar{\gamma}^{\mu=2} = -\tilde{\gamma}_t$, of the system are introduced by

$$\gamma_t^{\mu} = B(t)^{\mu\nu}a^{\nu}, \qquad \bar{\gamma}_t^{\mu} = \bar{a}^{\nu}B^{-1}(t)^{\nu\mu}, \tag{86}$$

with the time-dependent Bogoliubov transformation

$$B(t)^{\mu\nu} = \begin{pmatrix} 1+n(t) & -n(t) \\ -1 & 1 \end{pmatrix}. \tag{87}$$

The annihilation operators has the properties

$$\gamma_t|0(t)\rangle = 0, \qquad \langle 1|\tilde{\gamma}^{\dagger} = 0. \tag{88}$$

In terms of the annihilation and creation operators, the semi-free hat-Hamiltonian reduces to

$$\hat{H} = \omega\left(\gamma^{\dagger}\gamma_t - \tilde{\gamma}^{\dagger}\tilde{\gamma}_t\right) - i\kappa\left(\gamma^{\dagger}\gamma_t + \tilde{\gamma}^{\dagger}\tilde{\gamma}_t + 2\,[n(t)-\bar{n}]\,\gamma^{\dagger}\tilde{\gamma}^{\dagger}\right). \tag{89}$$

With this expression, we can see $\langle 1|\hat{H} = 0$, and can easily obtain the attractive expression of the solution for the Fokker-Planck equation as

$$|0(t)\rangle = \exp\left[[n(t)-n(0)]\,\gamma^{\dagger}\tilde{\gamma}^{\dagger}\right]|0\rangle. \tag{90}$$

This made us to imagine that the dissipative time evolution of the system is accompanied by the condensation of tilde and non-tilde particles. It may indicate that some kind of symmetry is spontaneously broken in producing the intrinsic entropy. This phenomena is called *the spontaneous creation of dissipation* [8, 9] which is still to be investigated from various view points.

The thermal space, the representation space of NETFD, is spanned by the basis vectors

$$\langle 1|\gamma(t)^m\, \tilde{\gamma}(t)^n, \qquad \gamma^{\natural}(t)^m\, \tilde{\gamma}^{\natural}(t)^n|0\rangle, \tag{91}$$

with $m, n = 0, 1, 2, 3, \cdots$. With the semi-free annihilation and creation operators, we can define the normal product, and produce a Wick-type formula leading to Feynman-type diagrams with the propagator:

$$G(t,t')^{\mu\nu} = -i\langle 1|T\left[a(t)^{\mu}\bar{a}(t')^{\nu}\right]|0\rangle = \left[B^{-1}(t)\mathcal{G}(t,t')B(t')\right]^{\mu\nu}, \tag{92}$$

with

$$\mathcal{G}(t,t')^{\mu\nu} = -i\langle 1|T\left[\gamma(t)^{\mu}\bar{\gamma}(t')^{\nu}\right]|0\rangle = \begin{pmatrix} G^R(t,t') & 0 \\ 0 & G^A(t,t') \end{pmatrix}, \tag{93}$$

where

$$G^R(t,t') = -i\theta(t-t')\mathrm{e}^{(-i\omega-\kappa)(t-t')}, \qquad G^A(t,t') = i\theta(t'-t)\mathrm{e}^{(-i\omega+\kappa)(t-t')}. \tag{94}$$

9.4 Relation to Monte Carlo Wave-Function Method

Upon the unified formulation of NETFD, we may be able to give a further understanding of the quantum jump within the Monte Carlo wave-function method [38]-[40] and also of the quantum-state diffusion method [41]-[43].

Let us investigate the Fokker-Planck equation (24):

$$\frac{\partial}{\partial t}|0(t)\rangle = -i\hat{H}|0(t)\rangle, \tag{95}$$

in order to reveal the relation of NETFD to the Monte Carlo wave-function method, i.e. quantum jump simulation [38]-[40].

Let us decompose the hat-Hamiltonian (21) as

$$\hat{H} = \hat{H}_0 + \hat{H}_1, \tag{96}$$

with

$$\hat{H}_0 = \omega\left(a^{\dagger}a - \tilde{a}^{\dagger}\tilde{a}\right) - i\kappa\,(2\bar{n}+1)\left(a^{\dagger}a + \tilde{a}^{\dagger}\tilde{a}\right), \tag{97}$$

$$\hat{H}_1 = 2i\kappa\left[(\bar{n}+1)\,a\tilde{a} + \bar{n}a^{\dagger}\tilde{a}^{\dagger}\right] - 2i\kappa\bar{n}, \tag{98}$$

and consider an equation:

$$\frac{\partial}{\partial t}|0_0(t)\rangle' = -i\hat{H}_0|0_0(t)\rangle'. \tag{99}$$

Note that $\hat{H}_1$ contains cross terms between tilde and non-tilde operators. We see that $\hat{H}_0$ and $\hat{H}_1$ have the properties

$$\langle 1|\hat{H}_0 = -2i\kappa\,(2\bar{n}+1)\,\langle 1|a^{\dagger}a, \tag{100}$$

$$\langle 1|\hat{H}_1 = 2i\kappa\,(2\bar{n}+1)\,\langle 1|a^{\dagger}a. \tag{101}$$

Introducing the *wave-functions* $|\psi(t)\rangle$ and $|\tilde{\psi}(t)\rangle$ through

$$|0_0(t)\rangle' = |\psi(t)\rangle|\tilde{\psi}(t)\rangle, \tag{102}$$

we have from (99) Schrödinger equations of the form

$$\frac{\partial}{\partial t}|\psi(t)\rangle = -iH_0|\psi(t)\rangle, \tag{103}$$

and its tilde conjugate, where

$$H_0 = \omega a^\dagger a - i\kappa\,(2\bar{n}+1)\,a^\dagger a. \tag{104}$$

The Monte Carlo simulations for quantum systems are performed for the Schrödinger equation (103) [38]-[40].

The time generation due to the hat-Hamiltonian $\hat{H}_0$ does not preserve the normalization of the ket-vacuum, i.e. the normalized ket-vacuum $|0(t)\rangle$ evolves for the time inclement dt as

$$\begin{aligned}\langle 1|0_0(t+dt)\rangle' &= \langle 1|\left(1 - i\hat{H}_0 dt\right)|0(t)\rangle \\ &= 1 - dp(t),\end{aligned} \tag{105}$$

with

$$dp(t) = 2\kappa\,(2\bar{n}+1)\,n(t)dt. \tag{106}$$

The recipe of the quantum jump simulation is that, for a time increment dt,

1. When $dp(t) < \varepsilon$ with a given positive constant ε, the normalized ket-vacuum evolves as

$$|0(t)\rangle \longrightarrow |0_0(t+dt)\rangle = \frac{|0_0(t+dt)\rangle'}{1-dp(t)} = \frac{|\psi(t+dt)\rangle}{\sqrt{1-dp(t)}}\frac{|\tilde{\psi}(t+dt)\rangle}{\sqrt{1-dp(t)}}. \tag{107}$$

2. In the case $dp(t) > \varepsilon$, a quantum jump comes in:

$$|0_1(t+dt)\rangle = \frac{-i\hat{H}_1 dt|0(t)\rangle}{dp(t)}. \tag{108}$$

The time increment dt should be chosen as the condition $dp(t) \ll 1$ being satisfied.

Averaging the processes $|0_0(t)\rangle$ and $|0_1(t)\rangle$ with the respective probability $1 - dp(t)$ and $dp(t)$ (i.e. these ket-vacuums looks like satisfying a certain kind of stochastic Liouville equation):

$$|0(t+dt)\rangle = [1 - dp(t)]\,|0_0(t+dt)\rangle + dp(t)|0_1(t+dt)\rangle, \tag{109}$$

we can obtain the Fokker-Planck equation (95).

9.5 Fluid-Dynamical Regime

We may be able to attribute thermal processes in the fluid-dynamical regime to the existence of some kind of extended object in thermal vacuums. In this respect, an investigation has been started [37].

Now, following Kubo [44] and Zubarev [45], let us introduce a thermal vacuum state in the Shrödinger representation, for non-equilibrium systems in the hydrodynamic stage, by the generalized Bloch equation

$$\frac{\delta}{\delta F_m(\mathbf{x},t')}|0_S(t)\rangle_\epsilon = -\epsilon e^{\epsilon(t'-t)}\hat{P}_m(\mathbf{x},t'-t)|0_S(t)\rangle_\epsilon, \tag{110}$$

with $t \geq t'$, where

$$\hat{P}_m(\mathbf{x}, t) = \mathrm{e}^{i\hat{H}t}\hat{P}_m(\mathbf{x})\mathrm{e}^{-i\hat{H}t}. \tag{111}$$

The generalized Bloch equation (110) satisfies the causality in the sense that the thermal vacuum $|0_S(t)\rangle_\epsilon$ is determined by the thermodynamic parameters $F_m(\mathbf{x}, t')$ in the past time t' $(t \geq t')$. The functional derivative here in (110) should be interpreted in the sense that $\delta F_n(\mathbf{x}', t')/\delta F_m(\mathbf{x}, t) = \delta_{n,m}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$. The inner product of the thermal bra- and ket-vacuum is the partition function: $\langle 1|0_S(t)\rangle_\epsilon = Q(t)$.

Using to the mapping rules given in [1, 2], the thermal vacuum defined by (110) can be expressed as

$$|0_S(t)\rangle_\epsilon = |\exp\left[-\sum_m \int d\mathbf{x} B_{m,t}(\mathbf{x})\right]\rangle, \tag{112}$$

where

$$B_{m,t}(\mathbf{x}) = \epsilon \int_{-\infty}^{0} dt' \mathrm{e}^{\epsilon t'} F_m(\mathbf{x}, t+t') P_m(\mathbf{x}, t') = B^{\ell}_{m,t}(\mathbf{x}) + \Pi_{m,t}(\mathbf{x}), \tag{113}$$

with

$$\Pi_{m,t}(\mathbf{x}) = -\int_{-\infty}^{0} dt' \mathrm{e}^{\epsilon t'} \left[F_m(\mathbf{x}, t+t')\dot{P}_m(\mathbf{x}, t') + \frac{\partial F_m(\mathbf{x}, t+t')}{\partial t} P_m(\mathbf{x}, t')\right] \tag{114}$$

$$= -\int_{-\infty}^{0} dt' \mathrm{e}^{\epsilon t'} j^m(\mathbf{x}, t') X_m(\mathbf{x}, t+t'). \tag{115}$$

The thermal vacuum (112) is specified by the condition that

$$\langle P_m(\mathbf{x}, t)\rangle = \langle P_m(\mathbf{x}, t)\rangle_{t,\ell}, \tag{116}$$

where the average $\langle \cdots \rangle$ with respect to the non-equilibrium thermal vacuum is defined by

$$\langle A(t)\rangle = \lim_{\epsilon\to 0} Q^{-1}(t)\langle 1|A(t)|0_H(t)\rangle_\epsilon, \tag{117}$$

with the thermal vacuum in the Heisenberg representation:

$$|0_H(t)\rangle_\epsilon = \mathrm{e}^{i\hat{H}t}|0_S(t)\rangle_\epsilon. \tag{118}$$

The Heisenberg operator $A(t) = \mathrm{e}^{i\hat{H}t}A\mathrm{e}^{-i\hat{H}t}$, satisfies the Heisenberg equation of motion within NETFD [1, 2]: $dA(t)/dt = i[\hat{H}, A(t)]$.

In the derivation of the expression (115) from (114), one needs a long and cumbersome calculation with the knowledge of thermodynamics. The thermodynamic currents operator $j^m(\mathbf{x}, t)$ reduce to be the thermal flux, the viscous flux and the diffusional flux in the case of the ℓ-component system, for example. On the other hand, the thermodynamic forces $X_m(\mathbf{x}, t)$ represent $\nabla\beta(\mathbf{x}, t)$, $-\beta(\mathbf{x}, t)\nabla{:}\mathbf{v}(\mathbf{x}, t)$, and $-\nabla[\beta(\mathbf{x}, t)\mu_i(\mathbf{x}, t)]$.

We see that the thermal vacuum (112) satisfies the Shrödinger equation:

$$\frac{\partial}{\partial t}|0_S(t)\rangle_\epsilon + i\hat{H}|0_S(t)\rangle_\epsilon = \epsilon\left(\hat{\Pi}_t + \delta\hat{\Pi}_t\right)|0_S(t)\rangle_\epsilon, \tag{119}$$

with

$$\hat{\Pi}_t = \sum_m \int d\mathbf{x} \hat{\Pi}_{m,t}(\mathbf{x}), \tag{120}$$

where $\hat{\Pi}_{m,t}(\mathbf{x})$ is given by taking $\wedge$ of (114) or (115) with

$$\hat{j}^m(\mathbf{x},t) = \frac{1}{2}\left[j^m(\mathbf{x},t) + \tilde{j}^{m\dagger}(\mathbf{x},t)\right], \tag{121}$$

and

$$\begin{aligned}\delta\hat{\Pi}_t = \frac{-\epsilon^2}{2^4\cdot 3}\sum_{m_1}\sum_{m_2}\sum_{m_3}\int d\mathbf{x}_1\int d\mathbf{x}_2\int d\mathbf{x}_3\Big\{\left[\dot{B}_1B_2B_3 + \dot{B}_1B_2\tilde{B}_3^\dagger + \dot{B}_1\tilde{B}_2^\dagger\tilde{B}_3^\dagger + \tilde{B}_1^\dagger\tilde{B}_2^\dagger\dot{\tilde{B}}_3^\dagger\right] + \text{t.c.}\\ - 2\left[\left(B_1\dot{B}_2B_3 + B_1\dot{B}_2\tilde{B}_3^\dagger\right) + \text{t.c.}\right]\Big\} + [\text{higher order terms with respect to } B].\end{aligned} \tag{122}$$

The symbol t.c. indicates to take a tilde conjugation. We have introduced an abbreviation like $B_1 = B_{m_1,t}(\mathbf{x}_1)$ in (122). The right hand side of (119) represents a *symmetry breaking* effect due to the thermal processes. It may be much more vivid if we write down the time-evolution equation of the thermal vacuum in the Heisenberg representation:

$$\frac{\partial}{\partial t}|0_H(t)\rangle_\epsilon = \epsilon\left[\hat{\Pi}(t) + \delta\hat{\Pi}(t)\right]|0_H(t)\rangle_\epsilon, \tag{123}$$

with

$$\hat{\Pi}(t) = \sum_m \int d\mathbf{x}\hat{\Pi}_m(\mathbf{x},t), \quad \delta\hat{\Pi}(t) = e^{i\hat{H}t}\delta\hat{\Pi}_t e^{-i\hat{H}t} \tag{124}$$

where

$$\hat{\Pi}_m(\mathbf{x},t) = e^{i\hat{H}t}\hat{\Pi}_{m,t}(\mathbf{x})e^{-i\hat{H}t} = -\int_{-\infty}^{t} dt' e^{\epsilon(t'-t)}\hat{j}^m(\mathbf{x},t')X_m(\mathbf{x},t'). \tag{125}$$

Because of the existence of the thermal processes, the Heisenberg thermal vacuum (118) changes in time due to the right hand side of (123), a symmetry breaking term.

With the help of the mapping rules [1, 2], the thermal vacuum (118) can be expressed as

$$|0_H(t)\rangle_\epsilon = |\exp\left[-\sum_m\int d\mathbf{x}B_m(\mathbf{x},t)\right]\rangle, \tag{126}$$

with

$$B_m(\mathbf{x},t) = e^{i\hat{H}t}B_{m,t}(\mathbf{x})e^{-i\hat{H}t} = B_m^\ell(\mathbf{x},t) + \Pi_m(\mathbf{x},t), \tag{127}$$

where $\Pi_m(\mathbf{x},t)$ is given by (125) by replacing $\hat{j}^m(\mathbf{x},t)$ with $j^m(\mathbf{x},t)$.

9.6 Topological aspect

It would be an interesting view point if we succeed in a topological characterization of a representation space instead of the characterization by means of the Fock space.

In quantum field theory, we have the Fock space as a representaion space based on the assumption that one-particle state, as well as vacuum state, are stable. In other words, the state vectors are specified by a set of integers and zero, which is a manifestation of the particle picture in quntum theory.

However there is no stabel particle in thermal equilibrium state of a finite temperature or in far-from equilibrium state. Therefore, one-particle state is no more stable. Even vacuum state is unstable in these situations since an initial state specified by a thermal vacuum state can evolve in time approaching to a final state specified by other themal vacuum perpendicular to the initial one. Then, the Fock states may not provide a appropriate representation space.

As a vector in a representation space can be expanded in terms of a set of the Fock-state vectors, a vector specified by means of some topological nature may be expanded in terms of a set of topological state vectors which provide us with a topological representation space. The expansion series in terms of the set of topological states may be characterized when one specifies the way of linking among the topological states. This kind of topological expansion was performed in the characterization of a chaotic orbit in terms of a set of unstable periodic orbits consisting of a strange attractor [46].

9.7 Prospect

NETFD may serve a good tool for an operator analysis in quantum information theory (see the paper by Ohya and Suyari in this Proceedings for a quantum information theory).

We expect that the present unified framework of NETFD may open a new field of dissipative quantum field theory which will provide us with a deeper insight of nature from the stand point of *quantum coherence and dissipation.*

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References

[1] T. Arimitsu and H. Umezawa, Prog. Theor. Phys. **74** (1985) 429.

[2] T. Arimitsu and H. Umezawa, Prog. Theor. Phys. **77** (1987) 32.

[3] T. Arimitsu and H. Umezawa, Prog. Theor. Phys. **77** (1987) 53.

[4] T. Arimitsu, Phys. Lett. **A153** (1991) 163.

[5] T. Arimitsu, Lecture Note of the *Summer School for Younger Physicists in Condensed Matter Physics* [published in "Bussei Kenkyu" (Kyoto) **60** (1993) 491, written in English], and the references therein.

[6] T. Arimitsu, *A Canonical Formalism of Dissipative Quantum Systems —NETFD—* Condensed Matter Physics (Lviv, Ukraine) **4** (1994) 26.

[7] J. A. Crawford, Nuovo Cim. **10** (1958) 698.

[8] T. Arimitsu, M. Guida and H. Umezawa, Europhys. Lett. **3** (1987) 277.

[9] T. Arimitsu, M. Guida and H. Umezawa, Physica **A148** (1988) 1.

[10] T. Arimitsu, Mathematical Sciences [*Sūri Kagaku*], June (1990) 22, in Japanese.

[11] T. Arimitsu and N. Arimitsu, Phys. Rev. E **50** (1994) 121.

[12] T. Arimitsu, J. Pradko and H. Umezawa, Physica **A135** (1986) 487.

[13] J. Schwinger, J. Math. Phys. **2** (1961) 407.

[14] L. V. Keldysh, Sov. Phys. JETP **20** (1965) 1018.

[15] K. Chou, Z. Su, B. Hao and L. Yu, Phys. Rep. **118** (1985) 1.

[16] R. Kubo, M. Toda and N. Hashitsume, *Statistical Physics II* (Springer, Berlin 1985).

[17] R. Stratonovich, J. SIAM Control **4** (1966) 362.

[18] K. Ito, Proc. Imp. Acad. Tokyo **20** (1944) 519.

[19] T. Saito and T. Arimitsu, Modern Phys. Lett. B **6** (1992) 1319.

[20] C. W. Gardiner and M. J. Collett, Phys. Rev. A **31** (1985) 3761.

[21] T. Saito and T. Arimitsu, Mod. Phys. Lett. B **7** (1993) 623.

[22] L. Accardi, Rev. Math. Phys. **2** (1990) 127.

[23] R. L. Hudson and K. R. Parthasarathy, Commun. Math. Phys. **83** (1984) 301.

[24] R. L. Hudson and J. M. Lindsay, Ann. Inst. H. Poincarè **43** (1985) 133.

[25] K. R. Parthasarathy, Rev. Math. Phys. **1** (1989) 89.

[26] I. R. Senitzky, Phys. Rev. **119** (1960) 670.

[27] M. Lax, Phys. Rev. **145** (1966) 110.

[28] H. Haken, *Optics. Handbuch der Physik* vol. XXV/2c (1970), [*Laser Theory* (Springer, Berlin, 1984)], and the references therein.

[29] R. F. Streater, J. Phys. A: Math. Gen. **18** (1982) 1477.

[30] H. Hasegawa, J. R. Klauder and M. Lakshmanan, J. Phys. A: Math. Gen. **18** (1985) L123.

[31] N. Obata, Bussei Kenkyu **62** (1994) 62, in Japanese.

[32] N. Obata, RIMS Report (Kyoto) **874** (1994) 156.

[33] N. Obata, RIMS Report (Kyoto) (1994) in press.

[34] F. Shibata and N. Hashitsume, Phys. Soc. Japan **44** (1978) 1435.

[35] N. N. Bogoliubov, A. A. Lognov and I. T. Todolov, *Introduction to Axiomatic Quantum Field Theory* (Benjamin 1975).

[36] T. Arimitsu, Y. Sudo and H. Umezawa, Physica **A146** (1987) 433.

[37] T. Arimitsu, J. Phys. A: Math. Gen. **24** (1991) L1415.

[38] K. Mølmer, Y. Castin and J. Dalibard, J. Opt. Soc. Am. B **10** (1993) 524.

[39] H. J. Carmichael, *An Open Systems Approach to Quantum Optics*, Lecture Notes in Physics (Springer-Verlag, Berlin 1993).

[40] B. M. Garraway and P. L. Knight, Phys. Rev. A **49** (1994) 1266.

[41] N. Gisin and I. C. Percival, J. Phys. A **25** (1992) 5677.

[42] N. Gisin and I. C. Percival, J. Phys. A **26** (1993) 2233.

[43] N. Gisin and I. C. Percival, J. Phys. A **26** (1993) 2245.

[44] R. Kubo, M. Yokota and S. Nakajima, J. Phys. Soc. Japan **12** (1957) 1203.

[45] D. N. Zubarev, *Nonequilibrium Statistical Thermodynamics* (Consultants Bureau, New York 1974) Chap. 4.

[46] H. Yamazaki and T. Arimitsu, *A Topological Analysis of a chaotic orbit* (1995) in preparation to submit.